LITERATURE CITED

1. C. M. Vest and V. S. Arpaci, "Overstability of a viscoelastic fluid layer heated from below," J. Fluid Mech., 36, No. 3, 613-623 (1969).
2. J. A. Eltayeb, "Nonlinear thermal convection in an elasticoviscous layer heated from below," Proc. R. Soc. London, 350, No. 1685, 161-176 (1977).
3. F. A. Garifullin, "Stability of a viscoelastic fluid layer heated from below," Zh. Prikl. Mekh. Tekh. Fiz., No. 6, 55-62 (1974).
4. K. Stewartson and J. T. Stuart, "A nonlinear theory for a wave system in plane Poiseuille flow," J. Fluid Mech., 48, No. 3, 529-545 (1971).
5. L. M. Hocking, S. Stewartson, J. T. Stuart, and S. N. Brown, "A nonlinear instability burst in plane parallel flow," J. Fluid Mech., 51, No. 4, 705-735 (1972).
6. A. Davey, L. M. Hocking, and K. Stewartson, "On the nonlinear evolution of three-dimensional disturbances in plane Poiseuille flow," J. Fluid Mech., 63, No. 3, 529-536 (1974).
7. C. Bruce and W. H. Schwarz, "An experimental study of second- and third-order fluids," Am. Chem. Soc., Polym. Prepr., 10, 152-159 (1969).

STABILITY OF THE FLOW OF A ROTATING LIQUID FILM ALONG THE INSIDE SURFACE OF A CYLINDRICAL TUBE

P. B. Begoulev and B. G. Froishteter

UDC 532.517

Under consideration is the problem of formation of Taylor vortices in a rotating film of a viscous incompressible fluid.

Recently, film flow of a rotating liquid has become used on a wide scale in diverse equipment of the chemical industry (evaporators, heat exchangers, chemical reactors). Superposition of rotation on gravity flow of a film facilitates uniform spraying of the equipment surface, especially at low rates of liquid consumption, and appreciably intensifies heat and mass transfer processes [1-3]. The principal parameter determining this intensification is the stream whirl factor numerically equal to the tangent of the angle between the line of flow and the generatrix of the tube. It has been established [2] that with $\tan\beta \sim 1$ a heat transfer coefficient more than twice as high as during gravity flow of the film is attainable. This intensification effect weakens as $\tan\beta$ decreases and, when $\tan\beta < 0.1$, it becomes negligible. Therefore, selection of the optimum tube height for imparting rotation to a film is one of the more important problems in rational equipment design. As is well known, under such conditions a film unwhirls along the height, because of friction at a solid surface, and $\tan\beta$ decreases correspondingly. This decrease along the tube height is not monotonic, however, and experiments have revealed [3] that at a certain spray density the $\tan\beta$ curve begins to break at some point, with the rate of change of $\tan\beta$ much higher along the initial segment than beyond this break point. This trend is illustrated graphically in Fig. 1: Experimental data are shown here obtained in another study [4] with flow of a water film along the surface of a tube $3\cdot10^{-2}$ m in diameter at a temperature of 19°C.

Exponential decreasing of $\tan\beta$ along the tube height has been established theoretically [3] and confirmed experimentally, as shown in Fig. 1. As to the break point and the corresponding change of the attenuation rate (at $\beta > \beta_{cr}$ the whirl factor decreases approximately 5.7 times faster), no satisfactory explanation of this phenomenon has yet been found. Meanwhile, determination of the critical $\tan\beta$ corresponding to the break point on a $\tan\beta = g(z/R)$ curve is of great practical importance, because maximum intensification of the transfer processes can be expected to occur within the initial range.

A break point on the $\tan\beta$ curve can be regarded as a consequence of a substantial change in the conditions of flow and, particularly, loss of stability so that a solution to the equations of laminar flow would not reveal it. Such a phenomenon is generally characteristic of flow of a liquid in the field of centrifugal forces. In the case of flow of a liquid between

Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 38, No. 6, pp. 1061-1065, June, 1980. Original article submitted April 12, 1979.

0022-0841/80/3806-0659$07.50

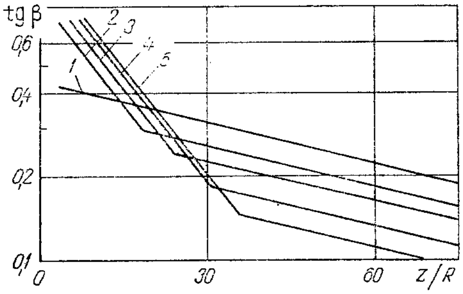

Fig. 1. Tangent of the film whirl angle as a function of the relative distance from the tube entrance, at various values of the Reynolds number $N_{Re,\ z}$: 1) 1000; 2) 2000; 3) 3000; 4) 4000; 5) 5000.

rotating coaxial cylinders, e.g., it is caused by breakdown of the laminar mode and formation of secondary streams in the form of toroidal vortices (Taylor vortices) [5]. It is reasonable to hypothesize that the appearance of vortices analogous to Taylor vortices in a rotating film can also be the cause of abrupt change of the transfer laws and, therefore, it seems worthwhile to analyze the stability of a rotating liquid film.

We will consider the flow of a rotating film of a viscous incompressible fluid along the inside surface of a cylindrical tube. The film stream is assumed to be axisymmetric so that the components of velocity of quiescent flow will be functions of r and z only

$$v_r^0 = v_r^0(r,\ z),\ v_z^0 = v_z^0(r,\ z),\ v_\varphi^0 = v_\varphi^0(r,\ z); \tag{1}$$

where $v_\varphi^0 \neq 0$ for a rotating film.

Let us examine the stability of film flow (1) against small axisymmetric perturbations, letting

$$\mathbf{v} = \mathbf{v}^0 + \mathbf{v}'(r)\exp\{i(\lambda z + pt)\}. \tag{2}$$

In deriving the equations of perturbed flow we will assume that the film thickness δ is much smaller than the tube radius R ($\delta \ll R$) and that the wavelength of perturbed flow is of the order of the film thickness δ. This assumption makes it permissible to disregard in the equations of perturbed flow all terms proportional to δ/R which contain the radial component of velocity and derivatives, with respect to z, of components of the velocity profile across the mainstream. One can then, without distorting the pattern of the phenomenon, replace expressions (1) with expressions for v_r^0, v_φ^0, and v_z^0 corresponding to steady flow of a rotating film

$$v_r^0 = 0,\ v_\varphi^0 = 3V\left(y - \frac{y^2}{2}\right),\ v_z^0 = 3U\left(y - \frac{y^2}{2}\right). \tag{3}$$

In this case the equations of perturbed flow in dimensionless variables can be written as

$$\{D^2 - a^2 - i\gamma - 3ia\,\mathrm{Re}_z f(y)\}(D^2 - a^2)\,v_r' - 3ia\,\mathrm{Re}_z v_r' = v_\varphi', \tag{4}$$

$$\{D^2 - a^2 - i\gamma - 3ia\mathrm{Re}_z f(y)\}\,v_\varphi' = -\,\mathrm{Ta}\,a^2 v_r'. \tag{5}$$

In the derivation of Eqs. (4)-(5) we have replaced the quantities v_φ^0/r, $(dv_\varphi^0/dr) + (v_\varphi^0/r)$ with their mean over the film thickness values according to relations (3).

As the boundary conditions for Eqs. (4)-(5) we stipulate adhesion

$$v_r' = Dv_r' = v_\varphi' = 0 \tag{6}$$

at the solid surface (y = 0) and continuity of normal stresses with zero shearing stresses at the free surface (y = 1). These conditions are

$$v_r' = D^2 v_r' = Dv_\varphi' = 0 \tag{7}$$

on the assumption that perturbations of the free surface can be disregarded for the solution of the problem of stability in the case of a rotating film.

With the aid of the well-known solution to an analogous problem of stability in the case of a thin liquid film heated in the gravitational field [5], one can demonstrate that

TABLE 1. Calculation of Results

Re_z	a	$-\gamma$	Ta	$tg\beta\sqrt{\frac{\delta}{R}}$
0	$0{,}2051\cdot10^{1}$	0	$0{,}691\cdot10^{3}$	
10	0,2051	$0{,}196\cdot10^{2}$	0,704	1,5310
20	0,2050	0,392	0,741	0,7855
30	0,2047	0,587	0,804	0,5453
40	0,2044	0,782	0,892	0,4310
50	0,2036	0,974	$0{,}101\cdot10^{4}$	0,3668
60	0,2029	$0{,}116\cdot10^{3}$	0,114	0,3248
70	0,2014	0,135	0,131	0,2984
80	0,1993	0,153	0,149	0,2785
90	0,1955	0,168	0,171	0,2652
100	0,1891	0,181	0,195	0,2549
200	0,4716	0,909	0,487	0,2014
300	0,5821	$0{,}170\cdot10^{4}$	0,826	0,1749
400	0,6633	0,259	$0{,}121\cdot10^{5}$	0,1587
500	0,7291	0,357	0,162	0,1464
600	0,7857	0,462	0,206	0,1381
700	0,8354	0,575	0,252	0,1304
800	0,8803	0,693	0,301	0,1251
900	0,9208	0,817	0,351	0,1201
1000	0,9589	0,946	0,404	0,1159
2000	$0{,}1337\cdot10^{2}$	$0{,}245\cdot10^{5}$	$0{,}101\cdot10^{6}$	0,0917
3000	0,1429	0,426	0,174	0,0802
4000	0,1581	0,629	0,255	0,0728
5000	0,1708	0,850	0,343	0,0676

boundary conditions (7) will be exact when $N_{Re,z} = 0$. Inasmuch as the presence of an axial velocity does not alter the mechanism of loss of stability by a rotating film, it can be assumed that adopting the boundary conditions (7) will not significantly affect the final results also when $N_{Re,z} \neq 0$.

Equations (4)-(5) with boundary conditions (6)-(7) constitute an eigenvalue problem. The flow will be stable or unstable depending on whether the imaginary part of γ is positive or negative. The condition Im $\gamma = 0$ defines the neutral line. For given values of a and $N_{Re,z}$, accordingly, it is necessary to find the real value of γ with which the Taylor number will also be real. The minimum value of N_{Ta} with respect to a is its critical value for a film.

The eigenvalue problem (5)-(7) is conveniently solved by the Galerkin method, with v_r' and v_φ' expanded into complete systems of functions satisfying the boundary conditions. There exist, in principle, many possible systems of functions into which v_r' and v_φ' can be expanded. In our case it is convenient to select for these functions simple polynomials in y

$$v_r' = \sum_{n=1}^{\infty} \alpha_n y^2 \left(y^2 - \frac{5}{2}y + \frac{3}{2}\right) y^{n-1}, \quad v_\varphi' = \sum_{n=1}^{\infty} \beta_n y (y-2) y^{n-1}, \tag{8}$$

just as in the Taylor problem of stability of flow between rotating coaxial cylinders.

Coefficients α_n and β_n are determined by the requirement that the errors in Eqs. (5)-(6) be orthogonal to the respective functions into which v_r' and v_φ' have been expanded. In practice in (8) only a finite number n = N terms are retained. This results in a system of N linear homogeneous equations for the series coefficients α_n and β_n. The necessary condition for the existence of a neutral solution is that the determinant of this system of equations be equal to zero. It has been demonstrated [7] that, as the Reynolds number $N_{Re,z}$ increases, more expansion functions are needed for a sufficiently accurate solution. Chandrasekhar has nevertheless shown [5] that a solution closely agreeing with experimental data can be obtained with the minimum number (N = 1) of expansion functions, if f(y) in Eqs. (5)-(6) is replaced with its mean over the gap width value $f(y) \equiv 1/3$. Inserting into Eqs. (5)-(6) the expansions (8) of v_r' and v_φ' with N = 1, also adding the requirement that the error in Eq. (5) be orthogonal to $u = y^2(y^2 - (5/2)y + 3/2)$ and the error in Eq. (6) be orthogonal to $v = y(y - 2)$, we obtain for α_1 and β_1 the system of equations

$$\alpha_1 (A - iC) = \beta_1 \langle uv \rangle, \quad \beta_1 (B - iD) = -\alpha_1 \mathrm{Ta} \langle uv \rangle a^2, \tag{9}$$

where

$$A = \langle D^4 uu \rangle - 2a^2 \langle D^2 uu \rangle + a^4 \langle uu \rangle; \quad B = \langle D^2 vv \rangle - a^2 \langle vv \rangle; \quad C = (\gamma + a\,\mathrm{Re}_z)(\langle D^2 uu \rangle - a^2 \langle uu \rangle) + 3\mathrm{Re}_z a \langle uu \rangle; \quad D = (\gamma + a\,\mathrm{Re}_z) \langle vv \rangle .$$

TABLE 2. Comparison of Theoretical and Experimental Data

Re_z	1000	2000	3000	4000	5000
$tg\,\beta_{cr}$ [3]	0,410	0,290	0,230	0,180	0,140
$tg\beta_{cr}$ Table 1	0,475	0,306	0,224	0,195	0,173

The condition for nontriviality of system (9) yields

$$\mathrm{Ta} = -\frac{(A-iC)(B-iD)}{a^2(\langle uv\rangle)^2}. \tag{10}$$

Since Im $\gamma = 0$ for the neutral line and N_{Ta} can be only real, we determine γ from the condition Im $N_{Ta} = 0$.

Inserting the expression for γ into expression (10) and minimizing N_{Ta} with respect to parameter α ($\alpha > 0$), we obtain the values of N_{Ta}, α, and γ which correspond to the neutral line at various values of $N_{Re,z}$ (Table 1). A comparison of theoretical data (Table 1) and experimental data on (tan β_{cr} at various values of $N_{Re,z}$ in Table 2 indicates an entirely satisfactory agreement between them.

One can conclude from the preceding analysis that the assumptions made here do not distort the physical pattern of the phenomenon but correctly reflect the quantitative as well as the qualitative aspects of the problem.

NOTATION

r, φ, z, cylindrical coordinates; v°, velocity of main flow; v', velocity of perturbed flow; λ is the wave number; p is the frequency; $i = \sqrt{-1}$; δ is the film thickness; R, tube radius; $y = (R - r)/\delta$; U, mean axial velocity; V, mean tangential velocity; tan $\beta = V/U$; $\alpha = \lambda\delta$; $\gamma = p\delta^2/\nu$; ν, kinematic viscosity; $N_{Re,z} = U\delta/\nu$; $N_{Re,\varphi} = V\delta/\nu$; $N_{Ta} = 3N_{Re,\varphi}^2\delta/R$; $D \equiv d/dy$; $f(y) = y - \frac{1}{2}y^2$; β, angle between the line of flow and the generatrix of the tube; and ⟨ ⟩ denotes averaging over the film thickness; t is time.

LITERATURE CITED

1. R. Z. Alimov, "Hydraulic drag and heat and mass transfer in a whirled stream," Teploenergetika, No. 3, 81-84 (1965).
2. G. I. Sergeev, "Intensity of convective heat transfer during whirled film flow," Teor. Osn. Khim. Tekhnol., 8, No. 5, 783-785 (1974).
3. I. M. Fedotkin, Yu. T. Borshchevskii, and V. S. Lipsman, "Effect of stream whirling on the flow of thin films," Teor. Osn. Khim. Tekhnol., 8, No. 3, 413-420 (1974).
4. I. M. Fedotkin and V. S. Lipsman, "Analysis of certain laws governing the hydraulics of film flow inside tubes," in: Food Industry [in Russian], 2nd ed. Tekhnika, Kiev (1971), pp. 78-83.
5. S. Chandrasekhar, Hydrodynamic and Hydromagnetic Stability, Oxford University Press, London (1961), p. 652.
6. E. Krueger and R. C. Di Prima, "The stability of a viscous fluid between rotating cylinders with axial flow," J. Fluid Mech., 19, No. 4, 528-544 (1964).
7. D. Wolovit, S. Tsiao, and R. C. Di Prima, "The stability of flow through an arbitrary gap between coaxial cylindrical surfaces with a radial temperature gradient," Trans. ASME, Appl. Mech., 31, No. 4, 12-21 (1964).